Q.P.Code: 23HS0835	R23	H.T.No.										

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR
(AUTONOMOUS)

B.Tech. II Year I Semester Regular Examinations February-2025

PROBABILITY AND COMPLEX VARIABLES

(Electronics & Communications Engineering)

Time: 3 Hours **Max. Marks: 70**

PART-A

(Answer all the Questions **10 x 2 = 20** Marks)

1	a	Define axiomatic definition of Probability.	CO1	L1	2M
	b	Write the density function of Rayleigh distribution.	CO1	L1	2M
	c	State Chebyshev's inequality.	CO2	L1	2M
	d	Define Statistical Independence of two r.v's X and Y	CO2	L1	2M
	e	Define joint characteristic function.	CO3	L1	2M
	f	Write any two properties of jointly Gaussian random variable.	CO3	L1	2M
	g	State Cauchy-Riemann (C-R) equations in cartesian coordinates.	CO4	L1	2M
	h	Show that the function $(x,y) = \sinh x \sin y$ is harmonic.	CO4	L2	2M
	i	Show that $\oint_C \frac{dz}{(z-a)} = 2\pi i$, where C is the circle $\lvert z-a\rvert = r$.	CO5	L2	2M
	j	State Cauchy Residue theorem.	CO5	L1	2M

PART-B

(Answer all Five Units 5 x 10 = 50 Marks)

UNIT-I

2		A manufacturing plant makes radios that each contain an integrated circuit (IC) supplied by three sources A, B and C. The probability that the IC in a radio came from one of the sources is $1/3$, the same for all sources. ICs are known to be defective with probabilities 0.001, 0.003, and 0.002 for sources A, B and C respectively. (i) What is the probability any given radio will contain a defective IC? (ii) If a radio contains a defective IC, find the probability it came from source A. Repeat for sources B and C.	CO1	L3	10M
		OR			
3	a	Spacecraft are expected to land in a prescribed recovery zone 80% of the time. Over a period of time, six spacecraft land. (i) Find the probability that none lands in the prescribed zone. (ii) Find the probability that at least one will land in the prescribed zone.	CO1	L2	5M
	b	Assume that the time of arrival of birds at a particular place on a migratory route, as measured in days from the first of the year, is approximated as a Gaussian random variable X with $a_X = 200$ days and $\sigma_X = 20$ days. (i) What is the probability the birds arrive after 160 days but on or before the 210^{th} day? (ii) What is the probability the birds will arrive after 231^{st} day?	CO1	L3	5M

UNIT-II

4		A random variable has a probability density $f_X(x) = \frac{5}{4}(1-x^4); \quad 0 < x \le 1$. Find (i) $E(X)$ (ii) $E(4X+2)$, (iii) $E(X^2)$ and (iv) $V(X)$	CO2	L1	10M

OR

5		Given the function $f_{X,Y}(x,y)=\frac{(x^2+y^2)}{8\pi}$; $x^2+y^2<b$. (i) Find a constant b so that this is a valid joint density function. (ii) Find $P\{0.5b<X^2+Y^2\leq 0.8b\}$. (Use polar coordinates in both parts).	CO2	L3	10M

UNIT-III

6		For two random variables X and Y have the joint density $f_{X,Y}(x,y)=x+y$; $0<x<1$ *and* $0<y<1$. Find: (i) $V(X)$ and $V(Y)$ (ii) The Covariance and (iii) Correlation coefficient (ρ)	CO3	L4	10M

OR

7		Gaussian random variables X_1 and X_2 for which $\overline{X_1}=2$, $\sigma_{X_1}^2=9$, $\overline{X_2}=-1$, $\sigma_{X_2}^2=4$ and $C_{X_1X_2}=-3$ are transformed to new random variables Y_1 and Y_2 according to $Y_1=-X_1+X_2$, $Y_2=-2X_1-3X_2$. Find (a) $E(X_1^2)$ (b) $E(X_2^2)$ (c) $\rho_{X_1X_2}$ (d) $\sigma_{Y_1}^2$ (e) $\sigma_{Y_2}^2$ and (f) $C_{Y_1Y_2}$	CO3	L3	10M

UNIT-IV

8		Prove that the function $f(z)$ defined by $f(z)=\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$;$(z\neq 0)$, $f(0)=0$ is continuous and Cauchy-Riemann equations are satisfied at the origin.	CO4	L3	10M

OR

9	a	Show that $u=\frac{1}{2}\log(x^2+y^2)$ is harmonic	CO4	L2	5M
	b	Find the analytic function whose real part is $e^{2x}(x\cos 2y-y\sin 2y)$	CO4	L4	5M

UNIT-V

10	a	Evaluate $\int_0^{1+i}(x^2-iy)\,dz$ along the line $y=x$.	CO5	L2	5M
	b	Evaluate $\oint_C \frac{e^{2z}}{(z-1)(z-2)}dz$, where C is the circle $\|z\|=3$ by using Cauchy's integral formula.	CO5	L3	5M

OR

11	a	Find the Laurent's expansion of $f(z)=\frac{1}{z^2-4z+3}$ for the region $1<\|z\|<3$	CO5	L3	5M
	b	Determine the poles of the function $f(z)=\frac{1}{(z+1)(z+3)}$ and the residue at each pole.	CO5	L3	5M

*** END ***